Optimal Control of Multi-particle Systems: Hamilton–Jacobi Equation in Eulerian and Lagrangian Settings

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Abstract

We study optimal control problems for interacting multi-particle systems, which naturally give rise to Hamilton–Jacobi equations in infinite-dimensional spaces and play a key role in mean-field models.

These problems can be formulated both in the Wasserstein space of probability measures (Eulerian viewpoint) and in a Hilbert space of parametrizations (Lagrangian viewpoint). The connection between the corresponding Hamilton–Jacobi theories has recently attracted significant attention.

Using tools from optimal transport, we prove that the Eulerian value function is a viscosity solution of a Hamilton–Jacobi–Bellman equation in Wasserstein space. Moreover, via a correspondence between Eulerian and Lagrangian formulations, we show that the Lagrangian value function is a viscosity solution of its Hamilton–Jacobi equation. This relies on recent advances in viscosity theory (cf. Jimenez–Marigonda–Quincampoix), with the key difference that, in our case, the Lagrangian value function and Hamiltonian arise naturally from an optimal control problem, rather than being constructed from the Eulerian objects.